Evaporating (2+1)-dimensional black strings

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Abstract

We investigate (2+1)-dimensional black strings in the Kaluza-Klein spacetime. The system is classically stable as long as the horizon size is much larger than the size of the compact space. Semiclassically, however, the horizon size shrinks gradually due to the energy loss through the Hawking radiation. Eventually, the system will enter into the regime of the Gregory-Laflamme instability and get destabilized. Subsequently, the spherically symmetric black hole is formed and evaporated in the usual manner. This standard picture may be altered by the dynamics of the internal space which induced by the Hawking radiation. We argue that the black string is excised from the Kaluza-Klein spacetime before the onset of the Gregory-Laflamme instability and therefore before the evaporation.

1 Introduction

In the conventional 4-dimensional spacetime, the black hole is considered as the blackbody which emits the Hawking radiation. Because of this radiation, the size of the horizon gradually shrinks and finally the black hole evaporates. Although there still exists debate on the final state after evaporation, the total picture is simple.

From the point of view of the super string theory, it is natural to consider higher dimensional spacetime. In this picture, the event horizon of the black hole would be the direct product of the usual event horizon and the compact internal space. It is interesting to investigate the evaporation process of the black hole in the Kaluza-Klein spacetime. In this paper, for simplicity, we consider the (2+1)-dimensional spacetime with S^1 as the compact internal space. The resulting Kaluza-Klein black hole is called the black string. The evaporation processes of the black strings could be different from the 4-dimensional one due to the Gregory-Laflamme instability [1]. It is known that the black string is unstable when the horizon radius is smaller than the scale of compactification. This instability changes the spacetime structure. The evaporation process taking into account this instability is depicted in the Figure 1. Soon after the onset of the Gregory-Laflamme instability, the spherically symmetric black hole would be formed. Subsequent evaporation process is very similar to the standard one.

Previously, we have considered the interplay between the radion and the Gregory-Laflamme instability [2, 3, 4]. There, the radion has played an important role. It is natural to expect the radion also makes a significant contribution to the evaporation process of the black strings. To the best of our knowledge, however, the interplay between the Hawking radiation and the radion dynamics has not been considered at all. Here, we study the role of the radion dynamics in the evaporation process of the (2+1)-dimensional black string.

The back reaction of the Hawking radiation could destabilize the radion. In fact, the energy of the Hawking radiation attracts the space around it. Thus, the radion is deformed by the Hawking radiation. In general, this deformation would be inhomogeneous. Hence, the spacetime is pinched at some radius. Consequently, assuming the singularity resolution, the black string might be excised from the spacetime. If this speculation is true, the radion dynamics would change the naive evaporation process completely. It should be stressed that the black string excision from the spacetime could occur before the onset of the Gregory-Laflamme instability and hence before the evaporation through the conventional process.

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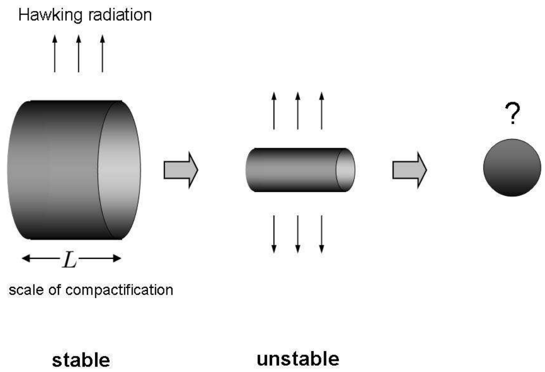

Figure 1: A naive picture of the evaporation process of the black string. Because of the Gregory-Laflamme instability, the black string becomes the spherically symmetric black hole.

2 (2+1)-dimensional Black Strings

In this section, we present our model and show the black string exists in this simple set up. We start with the (2+1)-dimensional dilaton gravity of the form

$$S[A,g] = M_3 \int d^3x \sqrt{-G}[AR^{(3)} + \frac{\lambda^2}{A}] \ , \tag{1}$$

where $G_{\mu\nu}$ is the 3-dimensional metric, $R^{(3)}$ is the 3-dimensional scalar curvature, A is the dilaton field, M_3 is the 3-dimensional Planck mass and λ is the parameter which have the mass dimension. The solution of the equation of motion obtained from the action (1) is

$$A = \lambda r \ , \quad ds^2 = -\alpha(r)dt^2 + \alpha(r)^{-1}dr^2 + dy^2 \ , \quad \alpha(r) \equiv \ln\left(\frac{r}{r_H}\right) \ , \tag{2}$$

where the period of the internal coordinate y is taken to be L. This solution represents the black string with horizon radius r_H and this spacetime is asymptotically flat because the curvature reads $R = 1/r^2$. It is known that the black string is unstable when $r_H \lesssim L$. The other way around, the black hole is stable as long as r_H is much larger than L. It is common to ignore the radion dynamics when we discuss this classical instability. However, once we take into account the semiclassical effect, we cannot ignore the radion dynamics. We need to study the evaporation process of this black string with the non-trivial radion dynamics.

3 Back Reaction of the Hawking Radiation

We consider quantum effect of the scalar field f in the black string background with the action

$$S[A,g,f] = M_3 \int d^3x \sqrt{-G}[AR^{(3)} + \frac{\lambda^2}{A}] + \int d^3x \sqrt{-G}[-\frac{1}{2}(\nabla f)^2] \ . \tag{3}$$

To take into account the radion dynamics, we parametrize the metric as

$$ds^2 = g_{ab}(x^a)dx^a dx^b + e^{-2\chi(x^a)}dy^2 \ , \tag{4}$$

where χ is the radion field. Here, we assume $r_H \gg L$ and all fields do not depend on y. Then, we can carry out the y integration in the action (3) to obtain

$$S[A,g,\chi,f] = M_3 L \int d^2x \sqrt{-g}\, e^{-\chi}[AR - 2\nabla A \cdot \nabla\chi + \frac{\lambda^2}{A}] + \int d^2x \sqrt{-g}\, e^{-\chi}[-\frac{1}{2}(\nabla f)^2] \ , \tag{5}$$

where R represents the 2-dimensional scalar curvature. We can study the evaporation process of the black string using the above effective 2-dimensional dilaton gravity. We treat the scalar field quantum mechanically, because we want to study the back reaction of the Hawking radiation. We define the effective action

$$W[A, g, \chi] = -i \ln \left(\int \mathscr{D}f \, \exp(i \int d^2x \sqrt{-g}\, e^{-\chi} [-\frac{1}{2}(\nabla f)^2]) \right) \tag{6}$$

and semiclassical energy momentum tensor

$$\langle T_{ab} \rangle = \frac{-2}{\sqrt{-g}} \frac{\delta W}{\delta g^{ab}} \ . \tag{7}$$

We can treat the back reaction perturbatively, that is, we regard the source T_{ab} as the small quantity. The master equation for the perturbed radion $\delta\chi$ is given by

$$-\Box \delta\chi - \frac{1}{A} \nabla A \cdot \nabla \delta\chi = \frac{1}{4M_3 L} \frac{1}{A} \langle T^a_a \rangle \ . \tag{8}$$

Note that the perturbed radion is gauge invariant as $\chi = 0$. Therefore, the gauge mode cannot appear in the master equation (8). Since the classical action of the matter field is Weyl invariant, the trace part of the energy-momentum tensor T^a_a should be zero. As is well known, however, Weyl symmetry has the anomaly in the quantum theory, namely,

$$\langle T^a_a \rangle = \frac{1}{24\pi} R = \frac{1}{24\pi r^2} . \tag{9}$$

Now, we can discuss the radion dynamics.

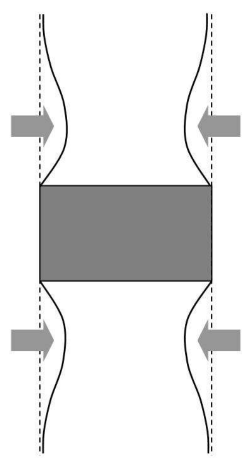

Figure 2: The radion has inhomogeneous profile due to the Hawking flux.

4 Excision of Black Strings

We set the initial conditions $\delta\chi(t=0,r) = \delta\chi_{,t}(t=0,r) = 0$. Then, we can deduce the radion dynamics for small t from master equation (8);

$$\delta\chi = \frac{1}{8M_3 L} \frac{\alpha(r)}{A} \langle T^a_a \rangle \, t^2 = \frac{1}{192\pi\lambda M_3 L} \frac{\ln(r/r_H)}{r^3} \, t^2 \ . \tag{10}$$

We gave a schematic picture of the radion dynamics in the Figure 2.

Now, we shall speculate the non-linear radion dynamics. The radion dynamics Eq.(10) can be understood from analogy to the cosmic dynamics. In cosmology, the matter tends to shrink the space

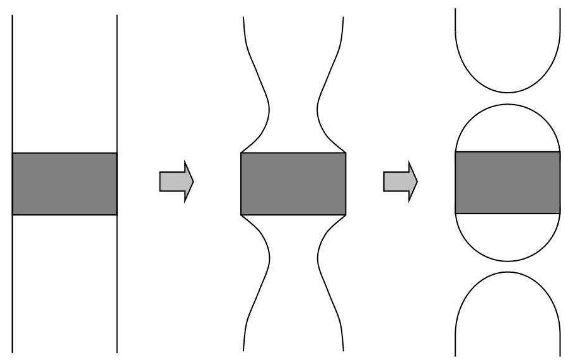

Figure 3: The speculative non-linear radion dynamics is shown. The black string is excised from the spacetime at the end of the day.

and, finally, makes the singularity. Similarly, the internal space may collapse due to the Hawking flux. Moreover, this occurs inhomogeneously. So we expect that the internal space is pinched off due to the Hawking radiation (Figure 3). Though the pinched point is classically singular, this is expected to be regularized by quantum effect. Thus, the black string region is excised from our spacetime. It looks like the evaporation of the black string from the observer outside. However, this evaporation process is completely different from the naive evaporation process in Figure 1.

5 Conclusion

We have investigated the (2+1)-dimensional black strings in the Kaluza-Klein spacetime. The solution of the master equation tells us that the internal space shrinks inhomogeneously (Figure 2). From this result, we can give the speculation for the non-linear radion dynamics. The black string may be excised from the spacetime due to the Hawking radiation (Figure 3). This can be interpreted as the evaporation of the black string from the observer outside. This evaporation process is different from that naively considered so far. Of course, at the present level of the analysis, we can not insist it strongly. However, it is worth to reconsider the evaporation process of black strings with taking into account the radion dynamics [5].

We need to analyze the non-linear radion dynamics to examine if our expectation in Figure 3 is correct. Furthermore, it is intriguing to consider the present issue in the context of the string theory.

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